Test 4

Logarithmic Functions & Continuous Random Variables
Semester One 2018
Year 12 Mathematics Methods
Calculator Assumed

PERTH MODERN SCHOOL
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Teacher:

_____ Mr McClelland
_____ Mrs. Carter
_____ Mr Gannon
_____ Ms Cheng
_____ Mr Staffe
_____ Mr Strain

Name:

Date: ______________ **7.45am**

You may have a calculator, a single-sided page of notes and a formula sheet for this section of the test.

Total__________/ **marks** **40 Minutes**

Questions 1 **(7 marks)**

Find the derivatives of the following. Do not simplify your answer.

(a) $\ln(2x^3 - 3x^2 + 4x - 1)^3$ (2 marks)

$$= \frac{3(2x^3-3x^2+4x-1)^2 \times (6x^2-6x+4)}{(2x^3-3x^2+4x-1)^3}$$ ✓ (chain Rule) ✓ $\left(\frac{d}{dx}\ln u = \frac{1}{u}\right)$

(b) $e^x \ln(x)$ (2 marks)

$$= e^x \ln x + e^x \frac{1}{x}$$ ✓ ✓ (product rule)

(c) $\ln(x)\cos(x) + \frac{\sin(x)}{x}$ (3 marks)

$$= \frac{1}{x}\cos x + \ln x(-\sin x) + \frac{\cos x \cdot (x) - \sin(x)}{x^2}$$ ✓ ✓

Question 2 **(~~4~~ 5 marks)**

(a) Use Polynomial Long division to simplify $\frac{x^2-2x+5}{x-3}$. (~~2~~ 3 marks)

$$\begin{array}{r} x+1 \\ x-3\overline{)\,x^2-2x+5} \\ \underline{x^2-3x} \\ x+5 \\ \underline{x-3} \\ 8 \end{array}$$

$$\frac{x^2-2x+5}{x-3} = (x+1) + \frac{8}{x-3}$$

(b) Hence find $\int \frac{x^2-2x+5}{x-3}\,dx$. (2 marks)

$$= \int (x+1)\,dx + \int \frac{8}{x-3}\,dx$$

$$= \frac{x^2}{2} + x + 8\ln(x-3) + C$$

-1 for missing "c"

Question 3 **(5 marks)**

(a) Find the constants a and b given that for $\{x \in \mathbb{R}: x \neq 2, x \neq -3\}$. (3 marks)

$$\frac{a}{x-2} + \frac{b}{x+3} = \frac{x+8}{x^2+x-6}$$

$ax + 3a + bx - 2b$

$$\frac{a(x+3)}{(x-2)(x+3)} + \frac{b(x-2)}{(x-2)(x+3)} = \frac{x+8}{(x-2)(x+3)}$$

x coeff. $\to a + b = 1$ ①

number coeff $\to 3a - 2b = 8$ ②

$2a + 2b = 2$ ③

② + ③ $= 5a = 10$

$a = 2$

$b = -1$

(b) Hence find $\int \frac{x+8}{x^2+x-6}\,dx$. (2 marks)

$$\int \frac{x+8}{x^2+x-6}\,dx = \int \frac{2}{x-2}\,dx - \int \frac{1}{x+3}\,dx$$

$$= 2\ln(x-2) - \ln(x+3) + C$$

$$= \ln\frac{(x-2)^2}{x+3} \;?$$

-1 for missing "C"

f.t previous "C"

Question 6 (5 marks)

The graph of the function with the rule $y = 3\log_2(x + 1) + 2$ intersects the axes at the points $(a,0)$ and $(0,b)$.

Find the exact values of a and b.

when $x = 0$ y-int

$y = 3\log_{10}(1) + 2$

$= \log_{10} 1^3 + 2(\log_{10} 2)$

$= \log_{10}(1 \times 4)$

$= \log_{10} 4$

$= 2\log_2 2$

$= 2 \Rightarrow (0,2)$

$\therefore b = 2$

when $y = 0$: x-int

$0 = 3\log_{10}(x+1) + 2$

$-2 = 3\log_{10}(x+1)$

$-\frac{2}{3} = \log_{10}(x+1)$

-0.7846

$\therefore 10^{-\frac{2}{3}} - 1 = x \Rightarrow a = 10^{-2/3} - 1$

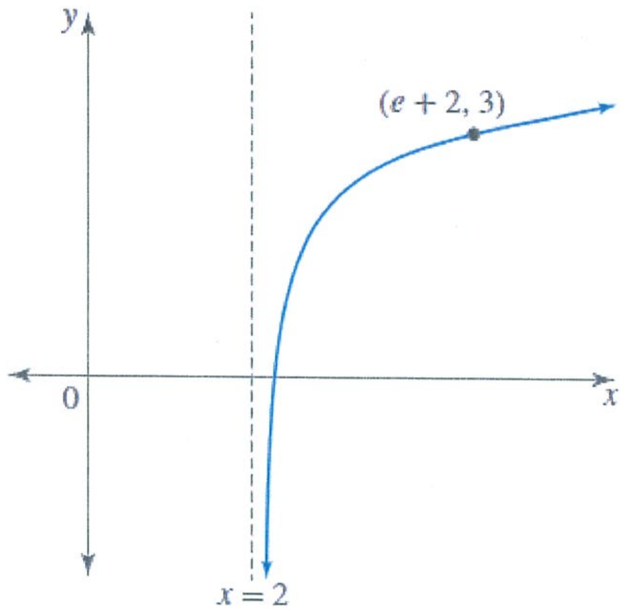
$100^{-\frac{1}{3}} - 1$

$\frac{1}{100^{\frac{1}{3}}} - 1$

$\frac{1}{10^{\frac{2}{3}}} - 1$

Question 4 **(2 marks)**

The rule for the function shown is $y = ln\,(x - m) + n$. Find the values of m and n.



$m = 2.$

$\ln(e+2-2) + n = 3$

$\ln e + n = 3$

$\therefore n = 2.$

Question 5 **(3 marks)**

Solve the following equations for x. Show full algebraic reasoning.

$$3e^{2x} - 5e^{x} - 2 = 0$$

$3\times(e^x)^2 - 5(e^x) - 2 = 0$

$1x \quad -2$
$3x \quad 1$

$(e^x - 2)(3e^x + 1) = 0$

$e^x = 2 \quad \therefore x = \ln 2.$

$e^x = -\frac{1}{3}$ (reject).

$\therefore x = \ln 2.$

Question 7 **(8 marks)**

There are two species of insects living in a suburb: the *Asla bibla* and the *Cutus pius*. The number of *Ala bibla* alive at time t days after 1 January 2000 is given by

$$N_A(t) = 10\,000 + 1000t, \qquad 0 \le t \le 15$$

The number of *Cutus pius* alive at time t days after 1 January 2000 is given by

$$N_C(t) = 8000 + 3 \times 2^t, \qquad 0 \le t \le 15$$

(a) (i) Show that $N_A(t) = N_C(t)$ if and only if $t = 3log_2 10 + \ log_2\left(\frac{2+t}{3}\right)$. (4 marks)

$$10000 + 1000t = 8000 + 3 \times 2^t$$

$$2000 + 1000t = 3 \times 2^t$$

$$\frac{2000 + 1000t}{3} = 2^t$$ ✓ (expression of 2^t)

$$\log\left(\frac{2000 + 1000t}{3}\right) = \log 2^t$$

$$\log\left(1000 \times \frac{2+t}{3}\right) = t \log 2$$ (factorising)

$$\frac{\log 1000 + \log \frac{2+t}{3}}{\log 2} = t$$ ✓ (expression of t in terms of log)

$$\frac{\log 10^3}{\log 2} + \frac{\log \frac{2+t}{3}}{\log 2} = t$$ ✓ (Apply $\log(A \times B) = \log A + \log B$)

$$\frac{3 \log 10}{\log 2} + \frac{\log \frac{2+t}{3}}{\log 2} = t$$

$$3 \log_2 10 + \log_2 \frac{2+t}{3} = t$$ ✓ (simplify using change-of-base)

Solve t

(ii) ~~Plot the graphs of $y = x$ and y~~ $= 3log_2 10 + log_2\left(\frac{2+t}{3}\right)$, ~~and find the coordinates of the point of intersection~~. (2 marks)

$(12.21, \boxed{12.21})$ ✓✓

(b) It is found by observation that the model for *Cutus pius* does not quite work. It is known that the model for the population of *Asla bible* is satisfactory. The form of the model for *Cutus pius* is $N_C(t) = 8000 + c \times 2^t$. Find the value of c, correct to two decimal places, if it is known that $N_A(15) = N_C(15)$. (2 marks)

$8000 + c \times 2^{15} = 10\,000 + 1000 \times 15$ ✓

$c \times 2^{15} = 17000$

$\therefore c = 0.52$ ✓